

# Mathematical Inverse Function (Equation) For Enzyme Kinetics

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## ABSTRACT

The most significant application of Lineweaver–Burk plot lies in the determination of Michaelis constant ( $K_m$ ), substrate concentration  $[S]$  at which the velocity ( $v$ ) of the enzyme catalyzed biochemical reaction attain half of its maximum ( $V_{max} \div 2$ ). For practical purposes,  $K_m$  is the concentration of substrate which allows the enzyme velocity to achieve half of its maximum  $V_{max}$  ( $V_{max} \div 2$ ). Most points on the plot are found far to the right of the y-axis. Through the reverse the mathematical steps and get inverse of substrate concentration ( $1 \div S$ ) back from some output value, say inverse of respective velocity ( $1 \div v$ ), one should undo each step in reverse order. It means that, one should subtract the inverse of maximum velocity ( $1 \div V_{max}$ ) from inverse of respective velocity ( $1 \div v$ ) and then multiply the result by  $\frac{V_{max}}{K_m}$ . This is going to yield the equation correspond to:  $\frac{1}{S} = \frac{V_{max}}{K_m} \times \frac{1}{v} - \frac{1}{K_m}$ . The  $1 \div S$  and  $1 \div v$  for given enzyme catalyzed biochemical reaction deserve symmetry, that is to say the symmetry between a Lineweaver Burk Plot (the real function) and the inverse function for enzyme kinetics of present attempt. The co-ordinates of the point of intersection of both the equations  $\frac{1}{v} = \frac{K_m}{V_{max}} \times \frac{1}{S} + \frac{1}{V_{max}}$  and  $\frac{1}{S} = \frac{V_{max}}{K_m} \times \frac{1}{v} - \frac{1}{K_m}$  correspond to:  $(\frac{1}{V_{max}-K_m}, \frac{1}{V_{max}-K_m})$ . This type of attempt may establish the baseline for classification of enzymes through mathematical principles.

**Keywords:** Baramati Constant;  $\frac{1}{V_{max}-K_m}$ ; mathematical approach

## I. INTRODUCTION

The velocity ( $v$ ) of biochemical reaction catalyzed by the enzyme vary according to the status of factors like: temperature; pH; enzyme concentration; substrate concentration; activators and inhibitors. There is no linear response of velocity ( $v$ ) of enzyme catalyzed biochemical reaction to the substrate concentration. This may be due to saturable nature of enzyme catalyzed biochemical reactions. If the initial velocity ( $v$ ) or rate of the enzyme catalyzed biochemical reaction is measured over a range of concentration of substrate  $[S]$ , it appears to increase. That is to say, initial velocity ( $v$ ) of the enzyme catalyzed biochemical reaction get increase according to the increase in the concentration of substrate  $[S]$ . This

tendency of increase in initial velocity ( $v$ ) of the enzyme catalyzed biochemical reaction according to the increase in the concentration of substrate  $[S]$  is observed upto certain level of the concentration of substrate  $[S]$ . At this concentration of substrate  $[S]$ , the enzyme exhibit saturation and exert the initial velocity ( $v$ ) of the enzyme catalyzed biochemical reaction to attain its maximum ( $V_{max}$ ). Hans Lineweaver and Dean Burk (1934) suggested the double reciprocal plot for presenting the data on substrate  $[S]$  and velocity ( $v$ ) of the enzyme catalyzed biochemical reaction. In enzyme kinetics, double reciprocal plot suggested by Hans Lineweaver and Dean Burk is well esteemed graphical presentation of the data on substrate concentration  $[S]$  and velocity ( $v$ ) of the enzyme catalyzed biochemical reaction recognized as, the “Lineweaver–Burk plot”. This plot

deserve wide applicability. The most significant application of Lineweaver–Burk plot lies in the determination of substrate concentration [S] at which the velocity (v) of the enzyme catalyzed biochemical reaction attain half of it's maximum ( $V_{\max} \div 2$ ). According to Lineweaver and Dean Burk (1934), substrate concentration [S] at which the velocity (v) of the enzyme catalyzed biochemical reaction attain half of it's maximum ( $V_{\max} \div 2$ ) is called as  $K_m$  (Michaelis constant). For practical purposes,  $K_m$  is the concentration of substrate which allows the enzyme velocity to achieve half of it's maximum  $V_{\max}$  ( $V_{\max} \div 2$ ). An enzyme with a high  $K_m$  has a low affinity for its substrate, and requires a greater concentration of substrate to achieve maximum  $V_{\max}$  (Keith J. Laidler, 1997).

The Lineweaver–Burk plot was widely used to determine important terms in enzyme kinetics, such as  $K_m$  and  $V_{\max}$ , before the wide availability of powerful computers and non-linear regression software. The y-intercept of such a graph is equivalent to the inverse of  $V_{\max}$ ; the x-intercept of the graph represents  $-1/K_m$ . It also gives a quick, visual impression of the different forms of enzyme inhibition. Accordingly, the mathematical equation suggested by Lineweaver and Dean Burk (1934) can be written as:

$$\frac{1}{V} = \frac{K_m}{V_{\max}} X \frac{1}{S} + \frac{1}{V_{\max}}$$

This Lineweaver–Burk plot deserve wide applicability. It is useful for the determination of  $K_m$ , the most significant factor in enzyme kinetics. The intercept on y – axis of Lineweaver–Burk plot is the inverse of  $V_{\max}$  that is ( $1/ V_{\max}$ ). The intercept on X – axis of Lineweaver–Burk plot is the inverse of  $-K_m$  that is ( $-1/K_m$ ). the x-intercept of the graph represents  $-1/K_m$ . This Lineweaver–Burk plot (double reciprocal plot) also gives a quick, visual impression of the different forms of enzyme activation and inhibition.

Hayakawa, et al (2006) reported the distortion of the error structure through the Lineweaver–Burk plot (double reciprocal plot). It is therefore, the

Lineweaver–Burk plot (double reciprocal plot) appears to be for the determination of enzyme kinetic parameters. Although it is still used for representation of kinetic data, (Hayakawa, et al ,2006) non-linear regression or alternative linear forms of the Michaelis–Menten equation such as the Hanes–Woolf plot or Eadie–Hofstee plot are generally used for the calculation of parameters (Greco and Hakala, 1979).

Dick (2011) explained type of enzyme inhibition. Accordingly, the Lineweaver–Burk plot (double reciprocal plot) can distinguish different types of inhibitors, such as competitive, non-competitive and uncompetitive inhibitors. The competitive inhibitors have the same y-intercept ( $1/ V_{\max}$ ) as uninhibited enzyme (since  $V_{\max}$  is unaffected by competitive inhibitors the inverse of  $V_{\max}$  also doesn't change) but there are different slopes and x-intercepts between the two data sets. The non-competitive inhibition produces plots with the same x-intercept ( $-1/K_m$ ) as uninhibited enzyme ( $K_m$  is unaffected) but different slopes and y-intercepts. Uncompetitive inhibition causes different intercepts on both the y- and x-axes (Berg, et al, 2002). John E. Dowd and Douglas Briggs (1965) reviewed the literature on “Estimates of Michaelis – Menten kinetic constants through the use of different linear transformation” and listed some problems with Lineweaver–Burk plot (double reciprocal plot). Accordingly, Lineweaver–Burk plot (double reciprocal plot) is classically used in older texts. It seems to be prone to the error. The y-axis takes the reciprocal of the rate of reaction – in turn increasing any small errors in measurement. Also, most points on the plot are found far to the right of the y-axis. This may be due to limiting solubility. It is not allowing for large values of  $1/[S]$  and hence no small values for [S]), calling for a large extrapolation back to obtain x- and y-intercepts.

## II. MATHEMATICAL INVERSE FUNCTION FOR ENZYME KINETICS

According to Hall, Arthur Graham and Frink, Fred Goodrich (1909), in mathematics, the equation (mathematical function) is represented in two forms: Real form and Inverse form. The inverse function may also called as anti – function. The inverse function is a function that "reverses" real form of function. If the function  $f$  applied to an input  $x$  gives a result of  $y$ , then applying its inverse function  $g$  to  $y$  gives the result  $x$ , and vice versa, i.e.,  $f(x) = y$  if and only if  $g(y) = x$  (Scheinerman, Edward R., 2013). For the purpose of fortified understanding of the concept of "Inverse Function", mathematics is citing simple example. Let us have a look on this example. Consider the real-valued function of a real variable given by  $f(x) = 5x - 7$  or  $y = 5x - 7$  (In " $y = m x + c$ " format. This is because, the original Lineweaver Burk plot deserve this format). Let us take a number  $x$ , multiply it by 5, then subtract 7 from the result, to reverse this and get  $x$  back from some output value, say  $y$ , one should undo each step in reverse order. It means that one should add 7 to  $y$  and then divide the result by 5. In functional notation this inverse function would be given by:  $g(x) = (1 \div 5)x + (7 \div 5)$  or  $y = (1 \div 5)x + (7 \div 5)$ .

Let us now have directly the equation for Lineweaver Burk plot:  $\frac{1}{v} = \frac{K_m}{V_{max}} X \frac{1}{S} + \frac{1}{V_{max}}$ .

The mathematical equation of Lineweaver Burk plot, itself is self explanatory. Inverse of substrate concentration ( $1 \div S$ ) is to be multiplied with the slope, that is  $K_m \div V_{max}$ . Finally, it is necessary to add the inverse of maximum velocity ( $1 \div V_{max}$ ) to get the value of inverse of respective velocity ( $1 \div v$ ) of enzyme catalyzed biochemical reaction. Of course, this is not the new idea of the present attempt. It has already been expected in the Lineweaver Burk plot of enzyme kinetics. The efforts of present attempt are concerned with getting the inverse function (equation) for Lineweaver Burk plot of enzyme kinetics.

The mathematical attempt for obtaining the inverse function for " $y = m x + c$ " is concerned with getting the value of " $X$ " back. To reverse the mathematical steps and get inverse of substrate concentration ( $1 \div S$ ) back from some output value, say inverse of respective velocity ( $1 \div v$ ), one should undo each step in reverse order. It means that, one should subtract the inverse of maximum velocity ( $1 \div V_{max}$ ) from inverse of respective velocity ( $1 \div v$ ) and then multiply the result by  $\frac{V_{max}}{K_m}$ . This is going to yield the equation correspond to:  $\frac{1}{S} = \frac{V_{max}}{K_m} X \frac{1}{v} - \frac{1}{K_m}$ .

For graphical presentation of equations of regular Lineweaver Burk plot:  $\frac{1}{v} = \frac{K_m}{V_{max}} X \frac{1}{S} + \frac{1}{V_{max}}$ ;

it is necessary to take the inverse of substrate concentration ( $1 \div S$ ) on  $X$  – axis and inverse of respective velocity ( $1 \div v$ ) of enzyme catalyzed biochemical reaction on  $Y$  – axis. This is going to serve the purpose in expectation of Lineweaver Burk plot:  $\frac{1}{v} = \frac{K_m}{V_{max}} X \frac{1}{S} + \frac{1}{V_{max}}$ .

For graphical presentation of equation:  $\frac{1}{S} = \frac{V_{max}}{K_m} X \frac{1}{v} - \frac{1}{K_m}$ ;

it is necessary to take the inverse of respective velocity ( $1 \div v$ ) of enzyme catalyzed biochemical reaction on  $X$  – axis and the inverse of substrate concentration ( $1 \div S$ ) on  $Y$  – axis. This is going to serve the purpose in expectation of Inverse function for Lineweaver Burk plot. That is to say plotting the readings of ( $1 \div S$ ) against ( $1 \div v$ ) and plotting the readings of ( $1 \div v$ ) against ( $1 \div S$ ) (Fig.1).

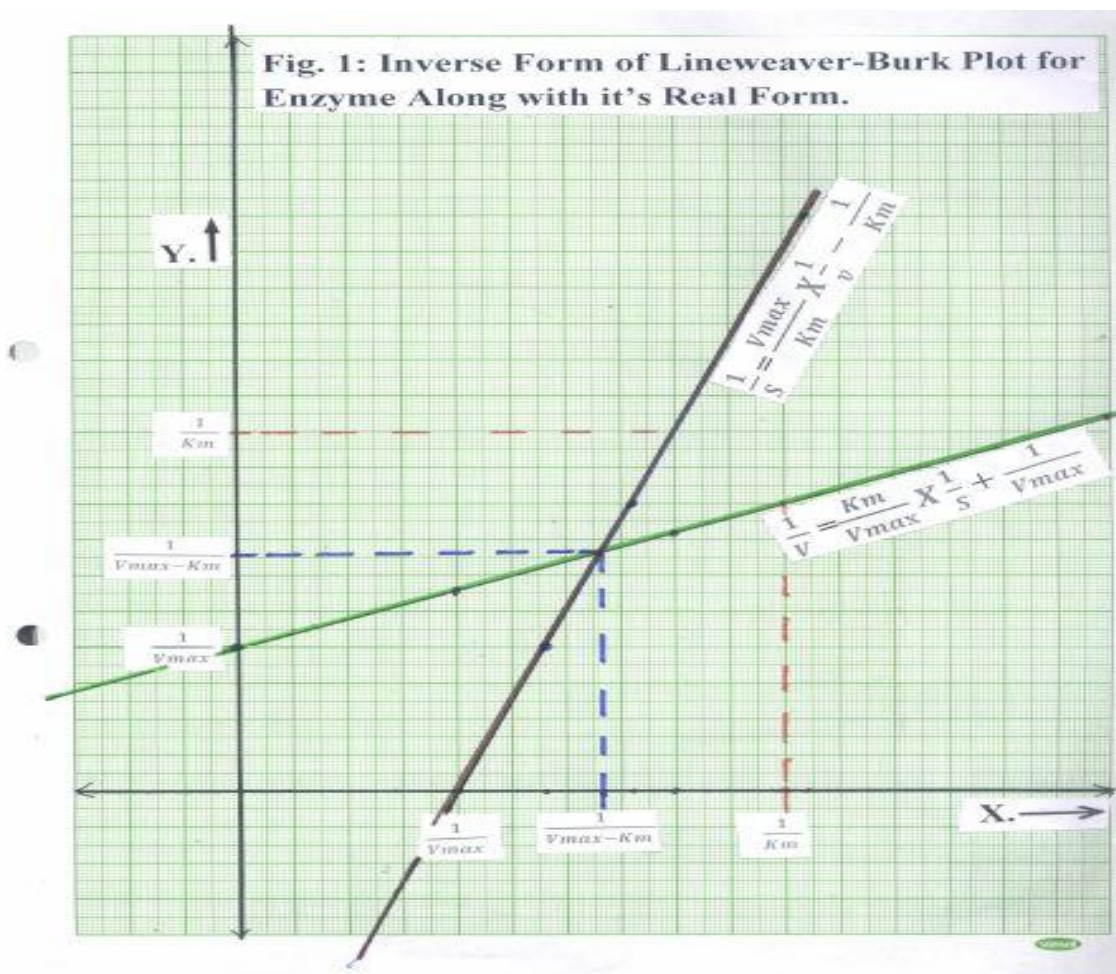


Figure 1

### III. PROPERTIES OF MATHEMATICAL INVERSE FUNCTION FOR ENZYME KINETICS

(A). The mathematical equation for regular Lineweaver Burk plot is explaining binary relation between the inverse of substrate concentration ( $1 \div S$ ) and inverse of respective velocity ( $1 \div v$ ) of enzyme catalyzed biochemical reaction. It is as good as a special type of mathematical binary relation. It means, many of the properties of inverse function for enzyme kinetics correspond to properties of converse relations.

(B). Theoretically, mathematical equation for regular Lineweaver Burk plot for a given enzyme is with unique inverse function. That is to say, for each concentration of substrate, there is a unique value for

velocity of enzyme catalyzed biochemical reaction. This follows since the inverse function must be the converse relation.

(C). If the  $1 \div S$  and  $1 \div v$  for given enzyme catalyzed biochemical reaction are inverses of each other, then the domain of  $1 \div S$  is equal to the range of  $1 \div v$  and the range of  $1 \div v$  is equal to the domain of  $1 \div S$ .

(D). The  $1 \div S$  and  $1 \div v$  for given enzyme catalyzed biochemical reaction deserve symmetry, then there is a symmetry between a function and its inverse. This statement is a consequence of the implication that for  $1 \div v$  to be invertible it must be bijective.

(E). The co-ordinates of the point of intersection of both the equations  $\frac{1}{v} = \frac{K_m}{V_{max}} X \frac{1}{S} + \frac{1}{V_{max}}$  and  $\frac{1}{S}$

$$= \frac{V_{max}}{K_m} X \frac{1}{v} - \frac{1}{K_m} \quad \text{correspond to: } \left( \frac{1}{V_{max}-K_m}, \frac{1}{V_{max}-K_m} \right).$$

(F). The  $1/v$  and  $1/S$  for given enzyme catalyzed biochemical reaction may deserve “one-to-one”. Even if a function  $1/S$  is not one-to-one relation with  $1/v$ ; it is possible to define an inverse through restricting the domain.

(G). The inverse function in enzyme kinetics is a function that reverses real form of mathematical equation for regular Lineweaver Burk plot. If the equation for  $(1/v)$ , applied to input  $(1/S)$ , gives a result of  $(1/v)$ . Then applying its inverse function  $(1/S)$  to  $(1/v)$  gives the result  $1/S$ , and vice versa.

(H). If  $(\frac{1}{S}, \frac{1}{v})$  is a point on the **graph** of the original equation of Lineweaver–Burk plot, then the point  $(\frac{1}{v}, \frac{1}{S})$  must be a point on the graph of the inverse function of enzyme kinetics. The Lineweaver–Burk plot and its inverse function are mirror images of each other with respect to the line  $y=x$ .

#### IV. APPLICATIONS OF MATHEMATICAL INVERSE FUNCTION (EQUATION) FOR ENZYME KINETICS

Applications of Interfacial enzyme catalyzed reactions are ubiquitous both in vivo and in technical applications. Analysis of kinetics of enzyme catalyzed reactions remains controversial. Michaelis–Menten equation expect reaching a steady-state condition in the opposite experimental limit, where the concentration of enzyme far exceeded the molar concentration of accessible surface sites. In particular, it is unclear whether conventional Michaelis–Menten theory. It requires a large excess of substrate. Extensive experimental study of the enzymatic hydrolysis of insoluble cellulose indeed proving that the conventional approach of Michaelis–Menten equation had a limited applicability. The interfacial

enzyme catalysis may reach a steady-state condition in the opposite experimental limit. In such situation, the concentration of enzyme far exceeded the molar concentration of accessible surface sites. Under such condition, an “inverse function or equation for enzyme kinetics” deserve applicability. Here it is essential to mention that, “inverse function or equation for enzyme kinetics” provides a general tool for kinetic analyses of interfacial enzyme reactions. It may establish theory provides a bridge to the accumulated understanding of steady-state enzyme kinetics. It reveals the density of enzyme attack sites on the substrate surface as probed by one specific enzyme.

The regular Michaelis–Menten equation  $(\frac{1}{v} = \frac{K_m}{V_{max}} X \frac{1}{S} + \frac{1}{V_{max}})$  explains the influence of concentration of substrate  $[S]$  on velocity of enzyme catalyzed biochemical reaction. Its inverse function may explain the influence of velocity of enzyme catalyzed biochemical reaction  $(v)$  on concentration of substrate  $[S]$ . That is to say, inverse function of the regular Michaelis–Menten equation and  $(\frac{1}{S} = \frac{V_{max}}{K_m} X \frac{1}{v} - \frac{1}{K_m})$  is going to explain the role of product of enzyme catalyzed biochemical reaction in controlling the rate of reaction. The regular Michaelis–Menten equation  $(\frac{1}{v} = \frac{K_m}{V_{max}} X \frac{1}{S} + \frac{1}{V_{max}})$  is demonstrating the  $K_m$  (Michaelis constant), the substrate concentration  $[S]$  at which the velocity  $(v)$  of the enzyme catalyzed biochemical reaction attain half of its maximum  $(V_{max} \div 2)$ . And ... and ... the inverse function is demonstrating the  $[1 \div (V_{max} - K_m)]$ , (Baramati Constant), point on both, the regular Michaelis–Menten equation  $(\frac{1}{v} = \frac{K_m}{V_{max}} X \frac{1}{S} + \frac{1}{V_{max}})$  and its inverse function  $(\frac{1}{S} = \frac{V_{max}}{K_m} X \frac{1}{v} - \frac{1}{K_m})$ . At this point  $[1 \div (V_{max} - K_m)]$ , (Baramati Constant), both the equations are equal with each other. This point  $[1 \div (V_{max} - K_m)]$ , (Baramati Constant) is going to serve the saturation of enzyme molecules and the substrate molecules in enzyme catalyzed biochemical reaction.

## V. CONCLUSION

Through the reverse the mathematical steps and to get inverse of substrate concentration ( $1 \div S$ ) back from some output value, say inverse of respective velocity ( $1 \div v$ ), one should undo each step in reverse order. It means that, one should subtract the inverse of maximum velocity ( $1 \div V_{max}$ ) from inverse of respective velocity ( $1 \div v$ ) and then multiply the result by  $\frac{V_{max}}{K_m}$ . This is going to yield the equation correspond to:  $\frac{1}{S} = \frac{V_{max}}{K_m} X \frac{1}{v} - \frac{1}{K_m}$ . The  $1 \div S$  and  $1 \div v$  for given enzyme catalyzed biochemical reaction deserve symmetry, then there is a symmetry between a function and its inverse. The co-ordinates of the point of intersection of both (Real form of Lineweaver Burk plot and it' Inverse form) the equations  $\frac{1}{v} = \frac{K_m}{V_{max}} X \frac{1}{S} + \frac{1}{V_{max}}$  and  $\frac{1}{S} = \frac{V_{max}}{K_m} X \frac{1}{v} - \frac{1}{K_m}$  correspond to:  $(\frac{1}{V_{max}-K_m}, \frac{1}{V_{max}-K_m})$ . The inverse function of the regular Michaelis-Menten equation and  $(\frac{1}{S} = \frac{V_{max}}{K_m} X \frac{1}{v} - \frac{1}{K_m})$  is going to explain the role of product of enzyme catalyzed biochemical reaction in controlling the rate of reaction. The  $[1 \div (V_{max} - K_m)]$ , (Baramati Constant), point on both, the regular Michaelis-Menten equation  $(\frac{1}{v} = \frac{K_m}{V_{max}} X \frac{1}{S} + \frac{1}{V_{max}})$  and it's inverse function  $(\frac{1}{S} = \frac{V_{max}}{K_m} X \frac{1}{v} - \frac{1}{K_m})$ . At this point  $[1 \div (V_{max} - K_m)]$ , (Baramati Constant), both the equations are equal with each other. This point  $[1 \div (V_{max} - K_m)]$ , (Baramati Constant) is going to serve the saturation of enzyme molecules and the substrate molecules in enzyme catalyzed biochemical reaction. The attempt on the inverse function for enzyme kinetics of present attempt may open a new chapter to classify the enzymes on the basis of mathematical approach.

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